STRICT IMPLICATION IN T

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In [6] we showed that the positive fragment of R is contained under suitable translation in the positive fragment of the system T of ticket entailment. There we expressed the view that T might be more closely related to E and R than it is to the (motivationally similar) system P-W. The results presented here are further evidence in support of that view. For the «same translations» used to embed $S4(S4_+)$ in $E(E_+)$ in [4] will embed $S4(S4_+)$ in $T(T_+)$. That this would be so was suggested by yet another similarity between T and E. If '$\Box A$' is defined in T^t as '$t \rightarrow A$', the resulting theory of modality is quite similar to that of E (which is itself very much like that of S4). The major difference between T^t-modality and E-modality is the unprovability in T^t of $\Box A \rightarrow A$.

The results of this paper also show that, just as with E and R (see [2] and [5]), the implications of some major non-relevant systems can be understood in T as kinds of implication. Theorem 2 below makes this claim obvious for strict implication à la S4. And given the results of [3], the same applies to intuitionistic implication.

In what follows we rely heavily on [4]. Much that is stated and proved there is assumed here.

Let $T(T_+)$ be formulated as in [1], and let $T^t(T^t_+)$ be the result of adding t to the vocabulary of $T(T_+)$ and adopting the new rules:

(R1) $\dfrac{\vdash A}{\vdash t \rightarrow A}$ (R2) $\dfrac{\vdash t \rightarrow A}{\vdash A}$

$S4(S4_+)$ can be (and is hereby) formulated by adding to $T(T_+)$

Ax0. $A \rightarrow (B \rightarrow B)$.

We *trivially* extend $S4(S4_+)$ to the system $S4^t(S4^t_+)$ by again adding t to the vocabulary and adopting the new axiom

Axl. $t \leftrightarrow (p \rightarrow p)$.

As in [4], we give the following definitions for T_+, T, $S4_+$, S4 where applicable:

D1. $f \leftrightarrow \overline{t}$

D2. $(A \supset B) \leftrightarrow ((A \& t) \rightarrow B)$

D3. $(A \dashv B) \leftrightarrow ((A \& t) \rightarrow (B \vee f))$.

Now define a translation $^+$ from $S4^t_+$ to T^t_+ as follows:

1. $A^+ = A$, for A atomic
2. $(B \& C)^+ = (B^+ \& C^+)$
3. $(B \vee C)^+ = (B^+ \vee C^+)$
4. $(B \rightarrow C)^+ = (B^+ \supset C^+)$.

And define a translation * from $S4^t$ to T^t thusly:

1. $A^* = A$, for A atomic
2. $(\overline{A})^* = \overline{A}^*$
3. $(B \& C)^* = (B^* \& C^*)$
4. $(B \vee C)^* = (B^* \vee C^*)$
5. $(B \rightarrow C)^* = (B^* \dashv C^*)$.

Proofs of the following lemmas are straightforward:

Lemma 1. $\vdash_{T^t_+} A^+$ iff $\vdash_{S4^t_+} A$

Lemma 2. $\vdash_{T^t} A^*$ iff $\vdash_{S4^t} A$.

We note only that Ax0 above does its job, and that Lemma 2 may be proved from right to left by using the admissible rule γ (essentially the rule of disjunctive syllogism), or adapting the argument of [4], pp. 189-92. For, E49-E55 of p. 191 (*et al.*) are theorems of and facts about T as well.

Since the exact translation of T^t_+ to T_+ given in [6] can be straightforwardly extended for T^t to T, and given Lemmas 1 and 2 above, we now have an exact translation from $S4^t$ ($S4^t_+$) to T (T_+). However, the more usual method of t-elimination can be used for our present purpose. In particular, let $p_1, \ldots, p_{n-1}$ be all the propositional variables occurring in A, and let p_n be the first propositional variable (in some assumed ordering thereof) that does not occur in A. Define t_A as $((p_1 \rightarrow p_1) \& \ldots \& (p_n \rightarrow p_n))$, and $A^\#$ as $A[t/t_A]$ (the proper substitution of t_A for t in A).

Our major results can then be stated as:

Theorem 1. $\vdash_{S4^t_+} A$ iff $\vdash_{T_+} A^{+\#}$

Theorem 2. $\vdash_{S4^t} A$ iff $\vdash_{T} A^{*\#}$.

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