SMILEY'S MATRICES AND DUNN'S SEMANTICS FOR TAUTOLOGICAL ENTAILMENT

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The following matrices, originally due to Smiley, are shown by Anderson and Belnap to be characteristic matrices for their system of tautological entailment E_{fde}: (1)

~	
4	1
2	2
3	3
1	4

v	1	2	3	4
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
4	1	2	3	4

&	1	2	3	4
1	1	2	3	4
2	2	2	4	4
3	3	4	3	4
4	4	4	4	4

→	1	2	3	4
1	1	4	4	4
2	1	1	4	4
3	1	4	1	4
4	1	1	1	1

The authors go on to remark that it is sometimes interesting to figure out «the intuitive sense» of such matrices but do not proceed to give any sense, intuitive or otherwise, to these matrices. It is my purpose in this paper to show that the semantics provided by Dunn for tautological entailment, when codified in a suitable matrix form, results in precisely these matrices for the truth functions «~» «v» and «&». The appropriate matrix for «→» may, however, be different from Smiley's thus giving a different characteristic set.

Dunn sets up his semantics with reference to the truth conditions under which certain «situations» would be true, allowing (in a way in which the ordinary two-valued matrices do not) for «p. ~ p» to have different truth conditions from «q. ~ q». (2) This is the heart of his «intuitive semantics». It is not within the scope of this paper to discuss the value of such a semantics as an explication of the notion of tautological entailment. That it does provide a semantics is undoub-

(1) ANDERSON and BELNAP, *Entailment*, 1975 p. 161.

(2) DUNN, 'Intuitive Semantic for First-Degree Entailment and «Coupled Trees»', *Philosophical Studies*, 1976, pp. 149-168. As Dunn shows that his semantics are isomorphic with those of the Routleys' «set-ups», the resulting matrices for the Routley's semantics would be the same. See Routley R. and Routley V. 'The Semantics of First Degree Entailment', *Nous*, 1972.

tedly established by Dunn by defining the notion of a relevance valuation as a *relation* from the set of sentences of E_{fde} to the set {T, F}. It is immediately apparent that such a relevance valuation will differ from orthodox valuations in that they are *functions*. Thus (p, T) and (p, F) may belong to the same valuation and neither may belong to a valuation.

In order to codify this semantics into matrix form it will be necessary to alter, albeit only formally, Dunn's presentation. I shall define a *relevance valuation* V for the set of sentences of E_{fde} in the following recursive manner. To each sentence A of E_{fde} is assigned a value, V(A), a sub-set of {T, F}, possibly null, so that

if A has the form ~B then	T ε V(A) iff F ε V(B) and
	F ε V(A) iff T ε V(B)
if A has the form B ∨ C then	T ε V(A) iff T ε V(B) or T ε V(C) and
	F ε V(A) iff F ε V(B) and F ε V(C)
if A has the form B & C then	T ε V(A) iff T ε V(B) and T ε V(C) and
	F ε V(A) iff F ε V(B) or F ε V(C).

For each relevance valuation, then, every sentence will receive one and only one of the values {T}, {T, F}, Φ and {F}. This allows the construction of the following three 'truth-tables':

~	
{F}	{T}
{T, F}	{T, F}
Φ	Φ
{T}	{F}

∨	{T}	{T, F}	Φ	{F}
{T}	{T}	{T}	{T}	{T}
{T, F}	{T}	{T, F}	{T}	{T, F}
Φ	{T}	{T}	Φ	Φ
{F}	{T}	{T, F}	Φ	{T, F}

&	{T}	{T, F}	Φ	{F}
{T}	{T}	{T, F}	Φ	{F}
{T, F}	{T, F}	{T, F}	{F}	{F}
Φ	Φ	{F}	Φ	{F}
{F}	{F}	{F}	{F}	{F}.

Replace «{T}» by «1», «{T, F}» by «2», «Φ» by «3» and «{F}» by «4» and the resulting matrices are those of Smiley given at the beginning of this paper.

There is a difference, however, when this method is extended to include «→». Dunn's intuitive semantics leads him to give what amounts to the following definition of *relevantly valid*: «A ⊢ B» is *relevantly valid* iff for every relevance valuation V, either T ∉ V(A) or T ε V(B). Since Dunn is able to prove that «A → B» is a theorem of E_{fde} iff «A ⊢ B» is relevantly valid, the appropriate matrix for this semantics would seem to be

→	{T}	{T, F}	Φ	{F}
{T}	{T}	{T}	{F}	{F}
{T, F}	{T}	{T}	{F}	{F}
Φ	{T}	{T}	{T}	{T}
{F}	{T}	{T}	{T}	{T}.

In this matrix the only reason for using «{T}» and «{F}» rather than «T» and «F» is uniformity with the other matrices so that a comparison with the Smiley matrices is possible. Replacing as before, the matrix for «→» is:

→	1	2	3	4
1	1	1	4	4
2	1	1	4	4
3	1	1	1	1
4	1	1	1	1.

With 1 taken as designated value the result is a set of characteristic matrices which differ from Smiley's only in the «→» matrix.

In order for complete agreement with Smiley's matrices, another definition of relevance validity must be given – one which Dunn aludes to but does not use: «A ⊢ B» is relevantly valid iff for every relevant valuation it is the case that both if T ε V (A) then T ε V (B) and if F ε V (B) then F ε V (A). Dunn indicates a proof that would show that these two definitions have the same extension. The appropriate matrix for the latter definition would be

→	{T}	{T, F}	Φ	{F}
{T}	{T}	{F}	{F}	{F}
{T, F}	{T}	{T}	{F}	{F}
Φ	{T}	{F}	{T}	{F}
{F}	{T}	{T}	{T}	{T}.

Replacing with numerals as before, the original Smiley matrix for «→» is obtained.

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