PREDICATE LOGIC WITHOUT PREDICATES

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Few philosophically oriented logicians seem to be aware of a rather interesting indexing or labeling technique which makes it possible to do predicate logic without predicates (and without quantifying over any domain of individuals) by the use of subscripts.

Let predicates be conceived of as being *listed in extension,* so that instead of the predicate P we have the (labeled) list (p_1) —, (p_2)—, ..., (p_i) —, ... of individuals to which the predicate applies. This extensional treatment of predicates presupposes, of course, that the predicate has (at least one) application. Thus we dispense with *P* in favor of a listing of the form (p_i) —, where i belongs to some index-set which need not of course be finite or even denumerable (e.g., we could employ real-number subscripts). The empty or universal predicate (which applies to everything) is supposed to correspond to the (labeled) list (u_1) —, (u_2) —, ... We shall adopt the notation that, whenever a predicate *P* with its corresponding list (p_1) —, (p_2) —,... is given, «$[p_j]$» is to denote the occupant of the j-th place in the p_i-listing, i.e., the item corresponding index (p_j). Further we adopt the rule *that every predicate list is of the same length as the list of the* u_i by the artificial device that the last «new entry» in the list is repeated *ad infinitum scilicet ad finem.* Thus if the universal (empty) predicate corresponds to the list,

(u_1) a, (u_2) b, (u_3) c, (u_4) d, (u_5) e,

then, if the predicate *P* corresponded in fact to the list,

(p_1) a, (p_2) c,

we would, under this convention, represent *P* as:

(p_1) a, (p_2) c, (p_3) c, (p_4) c, (p_5) c.

That is, since $[p_2]$ is the last «new» entry, we set $[p_i] = [p_2]$ for all $i > 2$.

The only pieces of (extra-propositional) logical machinery to be introduced are (i) quantification over subscripts [1], and (ii) the relation of (individual) identity.

Any statement of orthodox predicate logic can be transposed into this framework. For example «$(\exists x)$ Fx» becomes «$(\exists_i)$ $(\exists_j)$ $([u_i] = [f_j])$.»

[1] The listing for «universal predicate» serves to determine the domain over which the subscripting variables are to range.

and «(x) Fx» becomes «(i) (∃j) ($[u_i]$ = $[f_j]$).» (To preserve a distinction between necessary and contingent propositions, on such an approach, one must, however, adopt — as is in principle possible — a distinction between necessary and contingent identities (2). The standard categorical propositions can be handled as follows:

(A) All S is P: $(i)(\exists j)([s_i]=[p_j])$,
(E) No S is P: $(i)\sim(\exists_j)([s_i]=[p_j])$,
(I) Some S is P: $(\exists_i)(\exists_j)([s_i]=[p_j])$.
(O) Some S is not P: $(\exists_i)\sim(\exists_j)\ ([s_i]=[p_j])$.

It is an interesting characteristic of this scheme that all of the usual relationships embodied within the traditional «square of opposition» will obtain. A second noteworthy feature of this scheme is its capacity to accommodate the scholastic theory of *suppositio*, with its concept of suppositional descent from «All S is P» to «This S is P» (for us, «The i-th S is P») and from «This S is P» to «Some S is P.» As one commentator has remarked, this relationship among these three propositions escapes the approach customary in modern symbolic logic, because if «All S is P» is rendered as «(x) (Sx ⊃ Px)» and «Some S is P» as «(∃x) (Sx & Px),» then these, unlike their medieval counterparts, will «differ not just in quantification but also in internal structure.» (3) But just such a parallelism is inherent in our present treatment.

A comparable purely subscript-based treatment of relations is possible. Now instead of the linear list for a predicate we have a rectangular tabulation:

$$(r_{i0})\ —:\ (r_{i1})\ —,\ (r_{i2})\ —,\ \ldots,\ (r_{ij})\ —,\ \ldots\ i = 1, 2, \ldots$$

where (r_{ij}) is the label for the j-th individual that stands in the relation R to $[r_{i0}]$. Concretely if there are just four individuals a, b, c, d, and the relation R is such that a, b, and c each bears it to both the

(2) The analysis of a statement of the form «$[p_j]$= $[q_j]$» could be carried through in a manner parallel to Frege's analysis of «the morning star = the evening star,» and such an identity-statement would be classed as contingent if construed with reference to the extension (Fregean *Bedeutung*) of its terms, and as necessary if construed with reference to their intension (Fregean *Sinn*).

(3) G. B. Matthews, «Ockham's Supposition Theory and Modern Logic,» *Philosophical Review*, vol. 73 (1964), pp. 91-99. On the concept of supposition and its treatment by the subscripting technique see E. A. Moody, *Truth and Consequence in Medieval Logic* (Amsterdam, 1953), pp. 35-36 and 51-52, and R. G. Turnbull, «Ockham's Nominalistic Logic,» *The New Scholasticism*, vol. 36 (1962), pp. 313-329 (especially pp. 320-323).

other two and to these only, and d bears it to nothing, then we have the tabulation: (4)

(r_{10}) a : (r_{11}) b, (r_{12}) c, (r_{13}) c ,
(r_{20}) b : (r_{21}) a, (r_{22}) c, (r_{23}) c ,
(r_{30}) c : (r_{31}) a, (r_{32}) b, (r_{33}) b ,
(r_{30}) c : (r_{31}) a, (r_{32}) b, (r_{33}) b ,

Now the statement that R is anti-reflexive (« $\sim$ ($\exists$x) Rxx») becomes:

$$\text{(i)} \sim (\exists j)\ [(i \neq 0\ \&\ [r_{i0}] = (r_{ij}])].$$

The statement that R is symmetric («(x) (y) (Rxy $\rightarrow$ Ryx)») becomes:

$$\text{(i) (j)}\ [(\exists m)\ ([r_{im}] = [r_{j0}]) \rightarrow (\exists n)\ ([r_{jn}] = [r_{i0}])]$$

The statement «Rba» becomes «($\exists$j) ($[r_{20}] = [r_{1j}]$).» And in general all relational statements can be expressed with only our two items of supra-propositional logical machinery: quantification over subscripts and individual-identity.

It is clear that an approach of this kind should excite much sympathy from anyone committed to a nominalistic or an extensionalist point of view. (5) However the machinery that has been introduced is, *qua* machinery, strictly neutral as regards such philosophical commitments. For it would be possible to introduce into the picture non-existent individuals (non-designating singular terms) to serve as placeholders in labeled positions. With this — in principle perfectly feasible — step (6) (upon which we shall not elaborate here) one reintroduces all the complexities and perplexities which the extensionalists and nominalists seek to avoid.

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(4) It is an essential feature of the present example that the array is a square one. We apply to relations the same «stretching» device described for predicates in the second paragraph of the paper.

(5) A nominalist would (though an extensionalist need not) restrict labeling index-sets to sets that are finite (or perhaps denumerable ?).

(6) See T. HAILPERIN, and H. LEBLANC, «Nondesignating Singular Terms,» *The Philosophical Review*, vol. 68 (1959), pp. 239-243.